Project background. Estimating fish demography in the sea by trawling is a standard method of assessing the size and health of fish populations, and has been used for many decades now. The age of fish scales with its weight \( w \), or equivalently, size. Therefore, measuring the size allows to assess the age of fish, and demographers measure size distributions ranging from juvenile to adult fish. Under idealized equilibrium conditions, the size distribution of fish is a power law distribution, see Fig. 1 a). Such equilibrium distributions are useful in assessing the ‘health’ of a fish population, that is, to detect depletion of fish, e.g., due to disease or overfishing. An important question is therefore: how quickly does the fish population recover after perturbing it due, e.g., overfishing?

Figure 1: The size spectrum of a one-species-population approximately follows a power law distribution under equilibrium conditions [1, 2]. Size spectra resulting from Eq. (1) are shown (scaled to value one at the left side) as a function of individual size, \( w \), relative to asymptotic size, \( w_\infty \), for different growth laws \( g(w) \) (thick, thin, dashed).

Project description. A model for the size distribution can be formulated using size intervals, \([w, w + dw]\). Similar to mass conservation in fluid dynamics, one may set up a balance equation, see Figure 1 b): the abundance of a size group is a balance between how many individuals grow into the size group, how many grow out of it, and how many are dying. To solve the evolution of a size-structured population we must factor in the growth rate, because it sets the speed by which individuals move from one size class to the next. In a continuous size spectrum this balance is formalized in the so-called McKendrick-von Foerster partial differential equation:

\[
\frac{\partial}{\partial t} N(w, t) + \frac{\partial}{\partial w} \left[ g(w)N(w, t) \right] = -\mu(w)N(w) \tag{1}
\]

where \( g(w) \) is the growth rate (size per time) of individuals with mass \( w \) and \( \mu(w) \) the mortality rate (per time), and \( N(w, t) \) the size distribution (spectrum) at time \( t \). Boundary conditions vary depending on the situation.

Project aim. We aim at addressing the questions:

1. How stable are size distribution against perturbations (eigenvalue spectrum)?
2. How fast does a fish population restore to its equilibrium size distribution, \( N = N(w) \)?
3. How does dynamics between bony fish (the most common type) and elasmobranchs (sharks, rays, ...) differ?

The project includes the following tasks: i) Implement numerical solver for Eq. (1), analytical solutions and discuss their possible failure. ii) Investigate eigenvalue spectrum and determine stability of fish size distributions. iii) Classify impact of various perturbations (e.g., overfishing in a size class) on fish population.

References.
